MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2020

COMPUTER BASED TEST (CBT)

DATE : 03-09-2020 (SHIFT-2) | TIME : (3.00 pm to 6.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART-A : PHYSICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

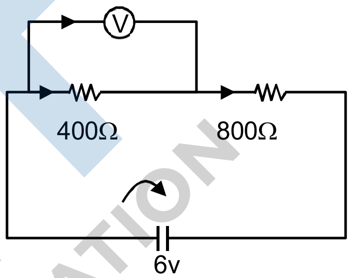
Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

1. Two resistors 400Ω and 800Ω are connected in series across a 6 V battery. The potential difference measured by a voltmeter of 10kΩ across 400Ω resistor is close to:

(1) 2 V (2) 1.95 V (3) 2.05 V (4) 1.8 V

Ans. (2)

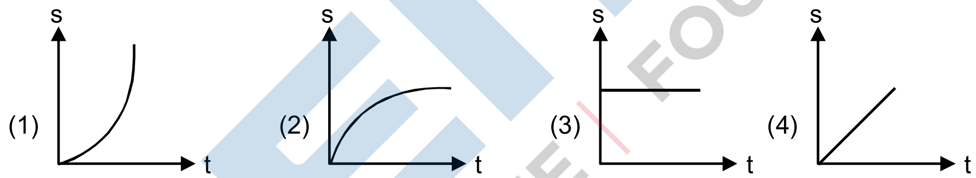
Sol. Let voltmeter reading is v



$$\frac{v}{400}\times 400+\left(\frac{v}{10000}+\frac{v}{400}\right)800=6$$

$$\Rightarrow \quad v+\frac{8v}{100}+2v=6\ ;\ \frac{77v}{25}=6\ ;\ v=\frac{150}{77}=1.95\ v$$

2. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) – time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale):



Ans. (1)

Sol. $P\cdot t=\frac{1}{2}mv^2 \Rightarrow V=\left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$

$$s=\int_0^t vdt=\sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt=\sqrt{\frac{2P}{m}}\cdot\frac{t^{3/2}}{3/2};\qquad S=\sqrt{\frac{8P}{9m}}\cdot t^{3/2}$$

3. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels a path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is:

(1) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1}-\frac{L_1}{n_2}\right)$ (2) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1}-\frac{L_2}{n_2}\right)$ (3) $\frac{2\pi}{\lambda}(n_1L_1-n_2L_2)$ (4) $\frac{2\pi}{\lambda}(n_2L_1-n_1L_2)$

Ans. (3)

Sol. The optical path between any two points is proportional to the time of travel.

The distance traversed by light in a medium of refractive index μ in time t is given by

d = vt .....(i)

where v is velocity of light in the medium. The distance traversed by light in a vacuum in this time,

$\Delta = ct$

$= c \times \frac{d}{v}$ [from equation (i)]

$= = d\frac{c}{v} = \mu d$ ....(ii) (Since, $\mu = \frac{c}{v}$)

This distance is the equivalent distance in vacuum and is called optical path.

Here, optical path for first ray $= n_1L_1$

Optical path for second ray $= n_2L_2$

Path difference $= n_1L_1 - n_2L_2$

Now, phase difference

$$= \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} \times (n_1L_1 - n_2L_2)$$

4. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take g = 10 m/s^2. Assume there is no rotational motion and loss of energy after the collision is negligible.]

(1) 21J (2) 23 J (3) 20 J (4) 19 J

Ans. (1)

Sol. Conservation of linear momentum

$0.1 \times 20 = (0.1 + 1.9) \times v$

$v = 1$ m/s

Using work energy theorem

$W_g = \Delta k$

$2 \times g \times 1 = k - \frac{1}{2} \times 2 \times 1^2$

$\therefore$ k = 21 J

5. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixture is 31°C. The value of 'm' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water = 1 cal g^{-1}°C^{-1})

(1) 2 (2) 4 (3) 3.2 (4) 2.6

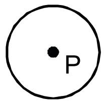
Ans. (1)

Sol. (200) (31 – 25) = m × 540 + m(1) (69)

1200 = m(609)

m = 2

6. A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is:



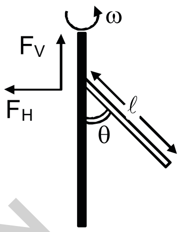
(1) much large than $|\vec{B}|$ but opposite to $\vec{B}$ (2) Zero

(3) much large than $|\vec{B}|$ but parallel to $\vec{B}$ (4) $\vec{B}$

Ans. (2)

Sol. When magnetic field is applied diamagnetic substance produces magnetic field in opposite direction so net magnetic field will be zero.

7. A uniform rod of length 'ℓ' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{m\ell^2}{12}\omega^2 \sin\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that:



(1) $\cos\theta = \frac{2g}{3\ell\omega^2}$ (2) $\cos\theta = \frac{3g}{2\ell\omega^2}$ (3) $\cos\theta = \frac{g}{2\ell\omega^2}$ (4) $\cos\theta = \frac{g}{\ell\omega^2}$

Ans. (2)

Sol. Torque of centrifugal force τ_{cf} = dm.xsinθω²xcosθ $= \frac{m}{\ell}\omega^2 \sin\theta\cos\theta\int_0^{\ell} x^2 dx$

$$\tau_{ef} = \frac{m\ell^2\omega^2 \sin\theta\cos\theta}{3}$$

$$\tau_{mg} = \tau_{cf}$$

$$mg\cdot\frac{\ell}{2}\sin\theta = \frac{m\ell^2\omega^2\sin\theta\cos\theta}{3}$$

$$\cos\theta = \frac{3g}{2\ell\omega^2}$$

8. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate dB/dt = 0.032 Ts^{-1}. The induced current in the loop is close to (Resistivity of the metal wire is 1.23×10^{-8} Ωm)

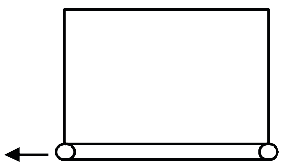
(1) 0.61 A (2) 0.34 A (3) 0.43 A (4) 0.53 A

Ans. (1)

Sol. Radius = d/2 = 2 mm

$$R_{wire} = \frac{\rho\ell}{A}$$

$$\phi = BA\ ; |e| = \frac{d\phi}{dt} = \frac{dB}{dt}(A)$$



$$i=\frac{e}{R}=\left|\frac{dB}{dt}\right|\frac{(A)^2}{\rho\ell}=\frac{0.032\times(\pi\times2\times10^{-3})^2}{1.23\times10^{-8}\times4\times0.3}=0.61\text{ A}$$

9. A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is:

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\sqrt{2}$ (4) 1

Ans. (2)

Sol. Remaining energy $\frac{1}{2}\left(\frac{1}{2}KA^2\right)=\frac{1}{2}KA'^2$

$$A'=\frac{A}{\sqrt{2}}$$

10. The radius R of a nucleus of mass number A can be estimated by the formula R = $(1.3\times10^{-15})A^{1/3}$ m. It follows that the mass density of n nucleus is of the order of: ($M_{prot.}=M_{neut.}\approx1.67\times10^{-27}$ kg)

(1) 10^{10} kg m^{-3} (2) 10^{24} kg m^{-3} (3) 10^{3} kg m^{-3} (4) 10^{17} kg m^{-3}

Ans. (4)

Sol. Theory Based

11. Concentric metallic hollow spheres of radii R and 4R hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference V9R) – V(4R) is:

(1) $\frac{Q_2}{4\pi\varepsilon_0R}$ (2) $\frac{3Q_2}{4\pi\varepsilon_0R}$ (3) $\frac{3Q_1}{16\pi\varepsilon_0R}$ (4) $\frac{3Q_1}{4\pi\varepsilon_0R}$

Ans. (3)

Sol. $V_{inner}=\frac{KQ_1}{R}+\frac{KQ_2}{4R}$

$V_{outer}=\frac{KQ_1}{4R}+\frac{KQ_2}{4R}$

Potential difference

$$\Delta V=V_{inner}-V_{outer}=\frac{3}{4}\cdot\frac{KQ_1}{R}=\frac{3}{16\pi\epsilon_0}\cdot\frac{Q_1}{R}$$

12. The electric field of a plane electromagnetic wave propagating along the x direction in vacuum is $\vec{E}=E_0\hat{j}\cos(\omega t-kx)$. The magnetic field $\vec{B}$, at the moment t = 0 is:

(1) $\vec{B}=\frac{E_0}{\sqrt{\mu_0\epsilon_0}}\cos(kx)\hat{k}$ (2) $\vec{B}=E_0\sqrt{\mu_0\epsilon_0}\cos(kx)\hat{k}$

(3) $\vec{B}=E_0\sqrt{\mu_0\epsilon_0}\cos(kx)\hat{j}$ (4) $\vec{B}=\frac{E_0}{\sqrt{\mu_0\epsilon_0}}\cos(kx)\hat{j}$

Ans. (2)

Sol. $B_0=\frac{E_0}{C}=\frac{E_0}{1/\sqrt{\mu_0\epsilon_0}}=E_0\sqrt{\mu_0\epsilon_0}$

As the light is propagating in x direction

& $\vec{E} \times \vec{B} \parallel \vec{C}$

$\therefore$ $\vec{B}$ should be in $\hat{k}$ direction

13. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to:

(1) 1 : 2 (2) 2 : 1 (3) 5 : 7 (4) 10 : 7

Ans. (2)

Sol. $qV = \frac{1}{2}mv^2$

Hence $v \propto \frac{1}{\sqrt{m}}$

$\frac{v_H}{v_{He}} = \sqrt{\frac{4}{1}} = 2:1$

14. If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is: Planck's constant h = 6.63 × 10^{-34} J.s.

Speed of light c = 3 × 10^8 m/s

(1) 1.1 eV (2) 1.5 eV (3) 2.0 eV (4) 3.1 eV

Ans. (4)

Sol. λ = 400 nm

Band gap $E_g = \frac{hc}{\lambda} = \frac{1237.5}{400} = 3.09\,eV$

15. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is:

(1) ML^2T^{-2} (2) ML^0T^{-3} (3) $M^2L^0T^{-1}$ (4) MLT^{-2}

Ans. (2)

Sol. Solar constant $= \frac{\text{Energy}}{\text{Time Area}}$

$= \frac{M^1L^2T^{-2}}{TL^2} = M^1L^0T^{-3}$

16. Two sources of light emit X-rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of the visible light of the given wavelengths is:

(1) 250 (2) $\frac{1}{500}$ (3) 500 (4) $\frac{1}{250}$

Ans. (2)

Sol. P_s – Power of source

$P_s = n\frac{hc}{\lambda}$; n = no. of photons emitted /s

$\Rightarrow$ $n \propto \lambda$ $\Rightarrow$ $\frac{n_2}{n_1} = \frac{\lambda_2}{\lambda_1} = 1/500$

17. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed?

(1) Multimeter shows a deflection, accompanied by a splash of light out of connected component in one direction and NO deflection on reversing the probes if the chosen component is LED

(2) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes is the chosen component is metal wire.

(3) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.

(4) Multimeter shows an equal deflection in both cases i.e., before and after reversing the probes if the chosen component is resistor.

Ans. (3)

Sol. Based on Theory

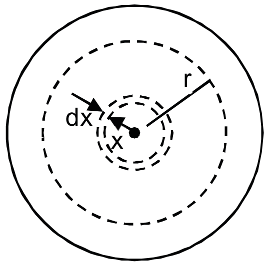
18. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$.

Then the gravitational field is maximum at:

(1) r = R (2) $r = \frac{1}{\sqrt{3}}$ (3) $r = \sqrt{\frac{5}{9}}R$ (4) $\sqrt{\frac{3}{4}}R$

Ans. (3)

Sol. $dm = \rho \times 4\pi x^2 dx = \rho_0\left(1 - \frac{x^2}{r^2}\right) \times 4\pi x^2 dx$



$$m = 4\pi\rho_0 \int_0^r \left(x^2 - \frac{x^4}{R^2}\right) dx\, s$$

$$m = 4\pi\rho_0 \left|\frac{r^3}{3} - \frac{r^5}{5R^2}\right|$$

$$E = \frac{Gm}{r^2} = \frac{G}{r^2} \times 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

$$E = 4\pi G\rho_0\left(\frac{r}{3} - \frac{r^3}{5R^2}\right)$$

E is maximum when $\frac{dE}{dr} = 0 \Rightarrow \frac{dE}{dr} = 4\pi G\rho_0\left(\frac{1}{3} - \frac{3r^3}{5R^2}\right) = 0$

$$\Rightarrow r = \frac{\sqrt{5}}{3}R$$

$$E_{max} = 4\pi G\rho_0 \times \frac{\sqrt{5}R}{3}\left[\frac{1}{3} - \frac{1}{5} \times \frac{5}{9}\right]$$

$$E_{max} = \frac{8\sqrt{5}}{27}\pi G\rho_0 . R$$

19. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)?

(1) 7 (2) 6 (3) 5 (4) 3

Ans. (2)

Sol. At constant pressure:

$\Delta Q = nC_P\Delta T \Rightarrow 160 = nC_P.50 \quad ...(1)$

At constant volume

$\Delta Q = nC_V\Delta T \Rightarrow 240 = nCV.100 \quad ...(2)$

Equation (1) divided by (2)

$$\frac{160}{240} = \frac{C_P}{C_V}\cdot\frac{50}{100}$$

$$\frac{C_P}{C_V} = \frac{4}{3} = 1 + \frac{2}{f}$$

f = 6

20. A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the nest 5 minutes will be close to:

(1) 33°C (2) 31°C (3) 35°C (4) 28°C

Ans. (1)

Sol. Using Newton's Law of cooling

$$\frac{50-40}{5\text{Min}} = K\left(\frac{50+40}{2} - 20\right) \quad ......(i)$$

Next 5 Min.

$$\frac{40-\theta}{5} = K\left(\frac{40+\theta}{2} - 20\right) \quad ......(ii)$$

Dividing (ii) / (i)

$$\frac{40-\theta}{10} = \frac{40+\theta-40}{50+40-40} = \frac{\theta}{50}$$

$$40 - \theta = \frac{\theta}{5}$$

$200 - 5\theta = \theta$

$$\therefore\ \theta = \frac{200}{6} = 33.3°C$$

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

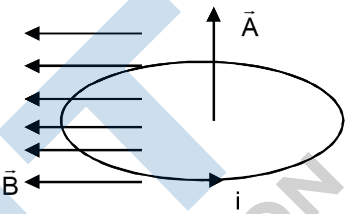
If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

21. A galvanometer coil has 500 turns and each turn has an average area of 3×10^{-4} m^2. If a torque of 1.5 Nm is required to keep this coil parallel to a magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is

Ans. 20

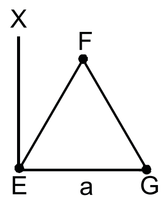


Sol. $\tau = |\vec{M} \times \vec{B}| = NiAB\sin(90°)$

$= NiAB = Ni\ S\ B$

$\Rightarrow \quad B = \frac{\tau}{NiS} = \frac{1.5}{500 \times 0.5 \times 3 \times 10^{-4}} = 20$

22. An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20}$ ma^2 where N is an integer. The value of N is



Ans. 25

Sol. $I = m \times O_2 + ma^2 + m\left(\frac{a}{2}\right)^2$

$= \frac{5}{4}ma^2 = \frac{25}{20}ma^2$

N = 25

23. A block starts moving up an inclined plane of inclination 30° with an initial velocity of υ_0. It comes back to its initial position with velocity $\frac{\upsilon_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$, The nearest integer to I is :

Ans. 346

Sol. A to B

$a_1 = g\sin30° + \mu g\cos30°$

$= \frac{g}{2} + \frac{\mu g\sqrt{3}}{2}$; g = 10 m/s^2

$V_0^2 - 2a_1(s) = 0$

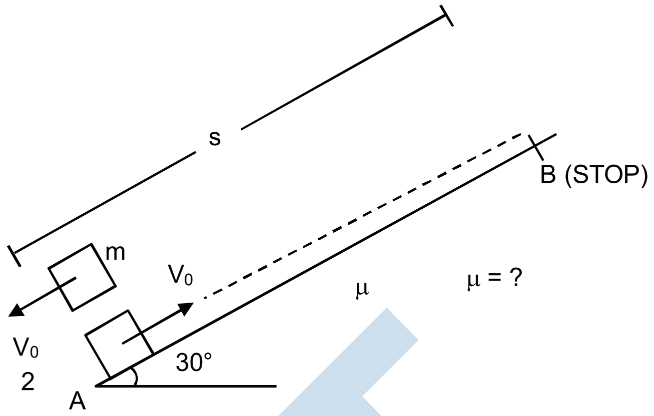
$s = \frac{V_0^2}{a_1}$ ....(i)

B to A

$a_2 = \frac{g}{2} - \frac{\mu\sqrt{3}}{2}g$

$\left(\frac{V_0}{2}\right)^2 = 2a_2(s)$

$s = \frac{V_0^2}{4a_2}$ ....(ii)



From equation (i) and (ii)

$\frac{V_0^2}{a_1} = \frac{V_0^2}{4a_2}$

$\Rightarrow a_1 = 4a_2$

$\Rightarrow 5 + 5\sqrt{3}\mu = 4(5 - 5\sqrt{3}\mu)$

$\Rightarrow 5 + 5\sqrt{3}\mu = 20 - 20\sqrt{3}\mu \Rightarrow 25\sqrt{3}\mu = 15 \Rightarrow \mu = \frac{\sqrt{3}}{5} = 0.346 = \frac{346}{1000}$

So, $\frac{I}{1000} = \frac{346}{1000}$

24. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10cm from the mirror. If the object is moved with a speed of 9 cms^{-1}, the speed (in cms^{-1}) with which image moves at that instant is

Ans. 1

Sol. Velocity of image

$v_i = -\frac{v^2}{u^2} \times v_0$

$= -\frac{10^2}{30^2}(9)$

$= -1$ cm/sec

25. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surroundings at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer?

Ans. 8791

Sol. $Q_2 = mL = 8000$

$\frac{w}{Q_2} = \frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2}$

$\frac{Q_1 - 8000}{8000} = \frac{27}{273}$

$Q_1 = 8791$

PART-B : CHEMISTRY

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

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26. The incorrect statement(s) among (a) – (b) regarding acid rain is (are):

(a) It can corrode water pipes

(b) It can damage structures made up of stone.

(c) It cannot cause respiratory ailments in animals

(d) It is not harmful for trees

(1) (a), (c) and (d) (b) (c) and (d) (c) (a), (b) and (d) (d) (c) only

Ans. (2)

Sol. (B) It is harmful for trees and plants

(C) It causes breathing problem in human being and animals

27. For the reaction 2A + 3B + $\frac{3}{2}$C $\rightarrow$ 3P, which statement is correct?

(1) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$ (2) $\frac{dn_A}{dt} = \frac{dn_B}{dt} = \frac{dn_C}{dt}$

(3) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$ (4) $\frac{dn_A}{dt} = \frac{3}{2}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

Ans. (3)

Sol. For a given reaction, $\text{rate} = -\frac{1}{2}\frac{dn_A}{dt} = -\frac{1}{3}\frac{dn_B}{dt} = -\frac{2}{3}\frac{dn_C}{dt}$

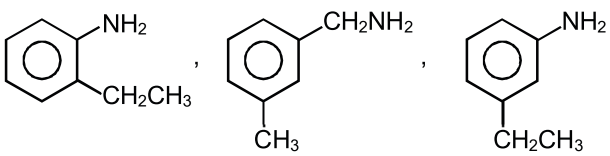
$$\text{rate} = \frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$$

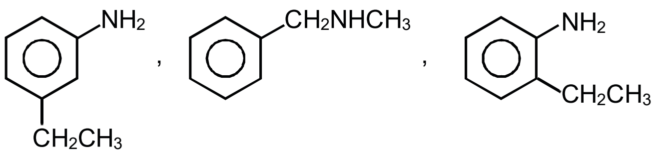
28. Three isomers A, B and C (mol. formula $C_8H_{11}N$) give the following results:

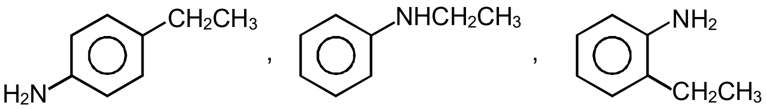
$$\text{A and C} \xrightarrow{\text{Diazotization}} P + Q \xrightarrow[\text{(ii) oxidation } (KMnO_4 + H^+)]{\text{(i) Hydrolysis}} \text{R (product of A)} + \text{S (product of C)}$$

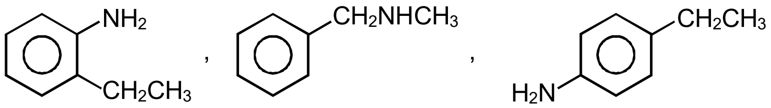
R has lower boiling point than S

$$B \xrightarrow{C_6H_5SO_2Cl} \text{alkali-insoluble product}$$ A, b and C, respectively are:

(1) 

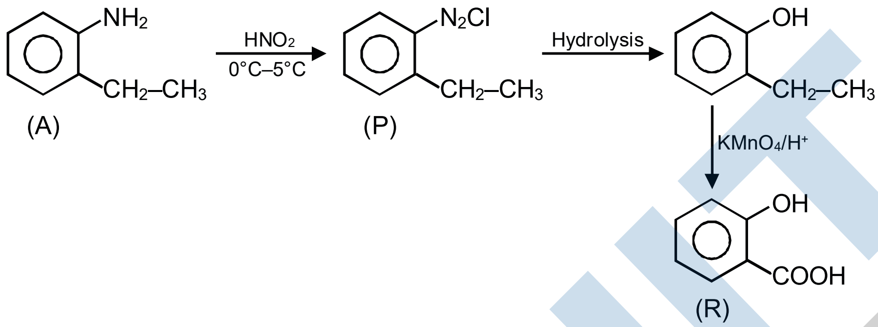
(2) 

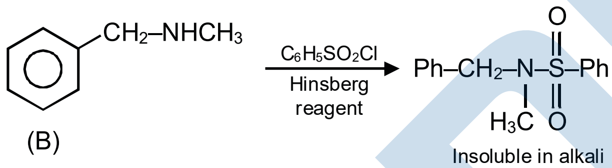
(3) 

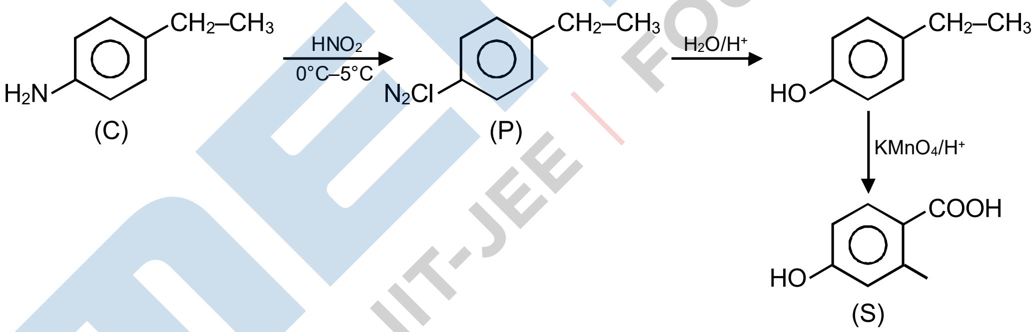
(4) 

Ans. (4)

Sol.







29. The five successive ionization enthalpies of an element are 800, 2427, 3658, 35024 and 32824 kJ mol^{-1}. The number of valence electrons in the element is:

(1) 4 (2) 3 (3) 5 (4) 2

Ans. (2)

Sol. As difference in 3rd and 4th ionisation energies is high so atom contains 3 valence electrons.

30. The increasing order of the reactivity of the following compounds in nucleophile addition reaction is:

Propanal, Benzaldehyde, Propanone, Butanone

(1) Benzaldehyde > Butanone < Propanone < Propanal

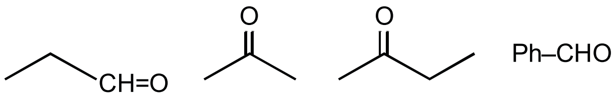
(2) Butanone < Propanone < Benzaldehyde < Propanal

(3) Benzaldehyde < Propanal < Propanone < Butanone

(4) Propanal < Propanone < Butanone < Benzaldehyde

Ans. (2)

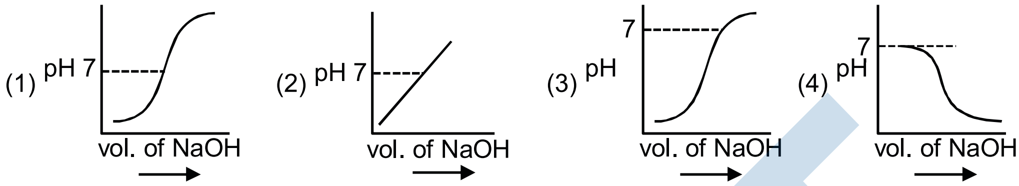
Sol. Rate of NAR α – I – M on substate



(1) (2) (3) (4)

1 > 4 > 2 > 3

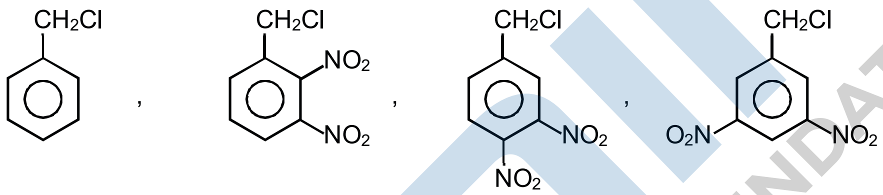
31. 100 mL of 0.1 M HCl is taken in a beaker and to it 100 mL of 0.1 M NaOH is added in steps of 2 mL and the pH is continuously measured. Which of the following graphs correctly depicts the change in pH?



Ans. (1)

Sol. At equivalence point pH is 7 and pH increases with addition of NaOH so correct graph is (1).

32. The decreasing order of reactivity of the following compounds towards nucleophilic substitution (S_N2) is:



(I) (II) (III) (IV)

(1) (II) > (III) > (I) > (IV)
(2) (II) > (III) > (IV) > (I)
(3) (III) > (II) > (IV) > (I)
(4) (IV) > (II) > (III) > (I)

Ans. (2)

Sol. S_N2 reaction depend upon –I, –M effect on substrate. On increase –I, –M, effect rate of S_N2 reaction increase.

33. Consider the hypothetical situation where the azimuthal quantum number, l, takes values 0, 1, 2, …….. n + 1, where n is the principal quantum number. Then, the element with atomic number:

(1) 6 has a 2p-valence subshell
(2) 9 is the first alkali metal
(3) 8 is the first noble gas
(4) 13 has a half-filled valence subshell

Ans. (3) [NTA answer is given (4)]

Sol. For n = 1 value of ℓ = 0, 1, 2
For n = 2 value of ℓ = 0, 1, 2, 3
So, according to n + l rule the filling order of subshells will be:

1s 1p 2s 1d 2p 3s 2d 3p 4s ……..

(1) 1st noble gas will have configuration $1s^2\ 1p^6$ so atomic number will be 8.

(2) 1st alkali metal will have electronic configuration $\Rightarrow 1s^1 \Rightarrow (Z = 1)$

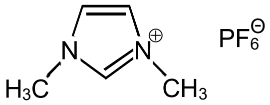
(3) Electronic configuration of C (Z = 6) $\Rightarrow 1s^2\ 1p^4$

(4) Z = 13, Electronic configuration = $1s^2\ 1p^6\ 2s^2\ 1d^3$

So it will not have half-filled electronic configuration.

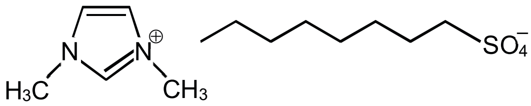
34. An ionic micelle is formed on the addition of:

excess water to liquid

(1) 

(2) liquid diethyl ether to aqueous NaCl solution

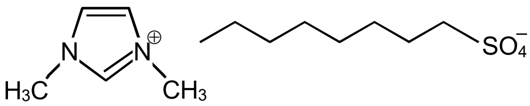
excess water to liquid

(3) 

(4) Sodium stearate to pure toluene

Ans. (3)

excess water to liquid

Sol. 

Due to presence of hydrophobic chain it forms micelle.

35. The d-electron configuration of $[Ru(en)_3]Cl_2$ and $[Fe(H_2O)_6]Cl_2$, respectively are:

(1) $t_{2g}^6e_g^0$ and $t_{2g}^4e_g^2$ (2) $t_{2g}^4e_g^2$ and $t_{2g}^4e_g^2$ (3) $t_{2g}^6e_g^0$ and $t_{2g}^6e_g^0$ (4) $t_{2g}^4e_g^2$ and $t_{2g}^6e_g^0$

Ans. (1)

Sol. $[Ru(en)_3]Cl_2 \Rightarrow Ru^{2+} = 4d^6\ t_{2g}^6, e_g^0$

$[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+} = 3d^6 = t_{2g}^4, e_g^2$

So, correct answer is (1).

36. The strengths of 5.6 volume hydrogen peroxide (of density 1 g/mL) in terms of mass percentage and molarity (M) respectively, are:

(Take molar mass of hydrogen peroxide as 34 g/mol)

(1) 1.7 and 0.5 (2) 0.85 and 0.5 (3) 1.7 and 0.25 (4) 0.85 and 0.25

Ans. (1)

Sol. For H_2O_2

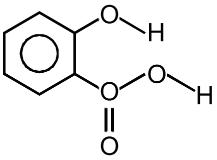
$$\text{Molarity} = \frac{\text{Volume strrength}}{11.2} = \frac{5.6}{11.2} = 0.5\text{ M}$$

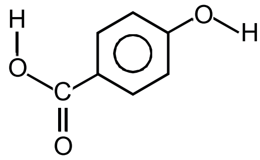
$$\text{Molarity} = \frac{\%(w/w) \times 10 \times d}{\text{GMM}}$$

$$0.5 = \frac{\%(w/w) \times 10 \times 1}{34}$$

$$\%(w/w) = \frac{0.5 \times 34}{10} = 1.7$$

37. Consider the following molecules and statements related to them:

(A) 

(B) 

(a) (B) is more likely to be crystalline than (A)

(b) (B) has higher boiling point than (A)

(c) (B) dissolves more readily than (A) in water

Identify the correct option from below:

(1) only (a) is true (2) (a) and (b) are true (3) (b) and (c) are true (4) (a) and (c) are true

Ans. (2)

Sol. Due to inter molecular H-Bonding in B, than A, B is more soluble and having more B.P point than A.

38. The incorrect statement is:

(1) Manganate and permanganate ions are tetrahedral

(2) Manganate and permanganate ions are paramagnetic

(3) In manganate and permanganate ion, the π-bonding takes place by overlap of p-orbitals of oxygen and d-orbitals of manganese

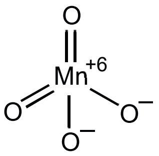
(4) Manganate ion is green in colour and permanganate ion is purple in colour

Ans. (2)

Sol.

Manganate

MnO_4^{2-}

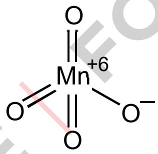


Paramagnetic, green in colour,

Tetrahedral & contains pπ-dπ bond

Permanganate

MnO_4^{1}



Diamagnetic, purple in colour,

Tetrahedral & contains pπ-dπ bond

39. Complex A has a composition of $H_{12}O_6Cl_3Cr$. If the complex on treatment with conc. H_2SO_4 loses 13.5% of its original mass, the correct molecular formulas of A is:

[Given : atomic mass of Cr = 52 amu and Cl = 35 amu]

(1) $[Cr(H_2O_6]Cl_3$

(2) $[Cr(H_2O)_4Cl_2]Cl \cdot 2H_2O$

(3) $[Cr(H_2O)_3Cl_3] \cdot 3H_2O$

(4) $[Cr(H_2O)_5Cl]Cl_2 \cdot H_2O$

Ans. (2)

Sol. Conc. H_2SO_4 acts as dehydrating agent.

Molar mass of given complex = 266.5 g/mol.

On treating with conc. H_2SO_4 the mass

lost by the complex $= \frac{13.5}{100}(266.5) \approx 36\,g$

= 2 moles of H_2O

Formula of the complex = $[Cr(H_2O)_4Cl_2]Cl \cdot 2H_2O$

40. A mixture of one mole each of H_2, He and O_2 each are enclosed in a cylinder of volume V at temperature T. If the partial pressure of H_2 is 2 atm, the total pressure of the gases in the cylinder is:

(1) 22 atm (2) 14 atm (3) 6 atm (4) 38 atm

Ans. (3)

Sol. $\boxed{P_{gas} = \dfrac{n_{gas}RT}{V}}$

as n, T & V constant So

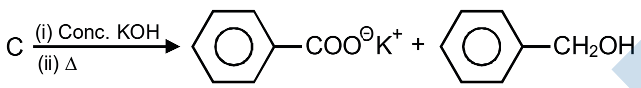
$P_{H_2} = P_{O_2} = P_{He} = 2 \text{ atm}$

So, $P_{Total} = P_{H_2} + P_{O_2} + P_{He} = 6 \text{ atm}$

41. The compound A in the following reactions is:

$A \xrightarrow[\text{(ii) Conc. } H_2SO_4/\Delta]{\text{(i) } CH_3MgBr/H_2O}$

$B \xrightarrow[\text{(ii) Zn/}H_2O]{\text{(i) } O_3} C + D$



$D \xrightarrow[\Delta]{Ba(OH)_2} H_3C-\underset{}{\overset{CH_3}{\overset{|}{C}}}=CH-\overset{O}{\overset{||}{C}}-CH_3$

(1) $C_6H_5-\overset{O}{\overset{||}{C}}-CH(CH_3)_2$

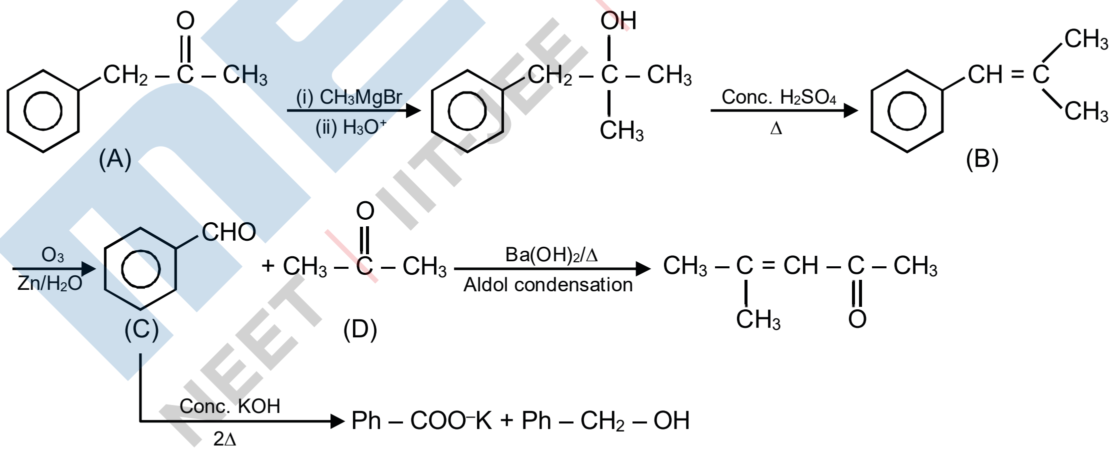
(2) $C_6H_5-CH_2-\overset{O}{\overset{||}{C}}-CH_3$

(3) $C_6H_5-\overset{O}{\overset{||}{C}}-CH_2CH_3$

(4) $C_6H_5-\overset{O}{\overset{||}{C}}-CH_3$

Ans. (2)

Sol.



42. Among the statement (I – IV), the correct ones are:

(I) Be has smaller atomic radius compared to Mg.

(II) Be has higher ionization enthalpy than Al.

(III) Charge/radius ratio of Be is greater than that of Al.

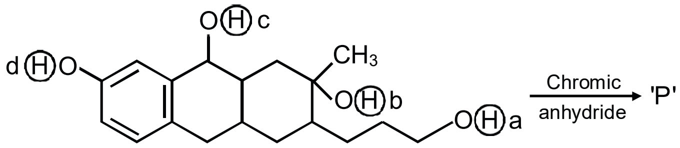
(IV) Both Be and Al form mainly covalent compounds.

(1) (II), (III) and (IV) (2) (I), (III) and (IV) (3) (I), (II) and (III) (4) (I), (II) and (IV)

Ans. (4)

Sol. Charge / radius ratio of Be and Al is same because of diagonal relationship. Remaining statements are correct.

43. Consider the following reaction:

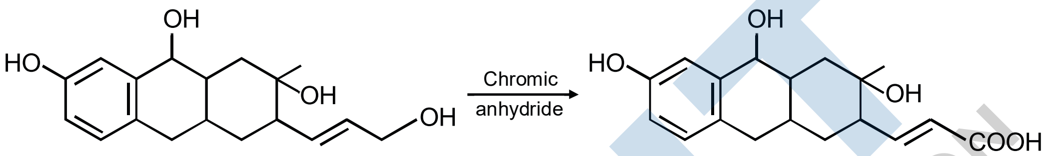


The product 'P' gives positive ceric ammonium nitrate test. This is because of the presence of which of these –OH group(s)?

(1) (b) only (2) (d) only (3) (b) and (d) (4) (c) and (d)

Ans. (1)

Sol.



Product 'P'

3° Alcohol gives Red colour with ceric ammonium nitrate

44. Match the following drugs with their therapeutic actions:

(i) Ranitidine (a) Antidepressant

(ii) Nardil (Phenelzine) (b) Antibiotic

(iii) Chloramphenicol (c) Antihistamine

(iv) Dimetane (Brompheniramine) (d) Antacid

(e) Analgesic

(1) (i)-(a); (ii)-(c); (iii)-(b); (iv)-(e);

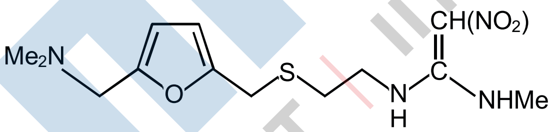
(2) (i)-(d); (ii)-(c); (iii)-(a); (iv)-(e);

(3) (i)-(d); (ii)-(a); (iii)-(b); (iv)-(c);

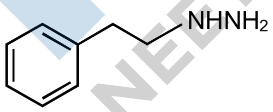
(4) (i)-(e); (ii)-(a); (iii)-(c); (iv)-(d);

Ans. (3)

Sol.

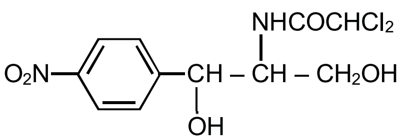


→ Antacid

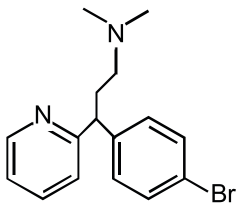


→ Antidepressant drugs

Phenelzine (Nardil)



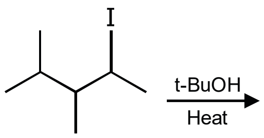
→ Antibiotics

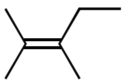


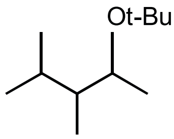
→ Antihistamine

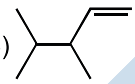
Brompheniramine
(Dimetapp. Dimetane)

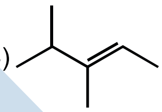
45. The major product in the following reaction is:



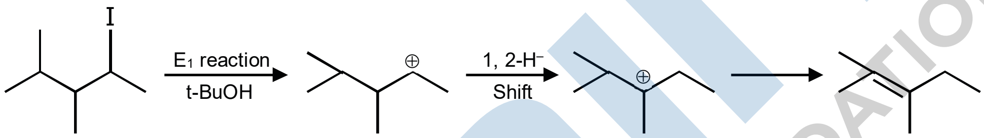
(1) 

(2) 

(3) 

(4) 

Ans. (1)

Sol. 

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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46. An acidic solution of dichromate is electrolyzed for 8 minutes using 2A current. As per the following equation $Cr_2O_7^{2} + 14H^+ + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$

The amount of Cr^{3+} obtained was 0.104 g. The efficiency of the process (in%) is (Take : F = 960000 C, At. mass of chromium = 52)

Ans. 60

Sol. Charge (q) = it = 2 × 8 × 60 = 960 C

$$\frac{960}{96000} = 0.01F$$

$$Cr_2O_7^{2} + 14H^+ + 6e^- \longrightarrow \quad 2Cr^{3+} \quad + \quad 7H_2O$$

$$0.01F \qquad \frac{1}{3} \times 0.01\,\text{mol}$$

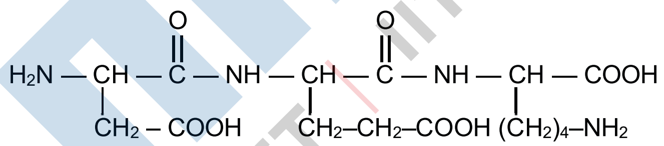
Theoretical mass of $Cr^{3+} = \frac{1}{3} \times \frac{600}{96000} \times 52 = 0.173\,g$

So, efficiency $= \frac{w_{actual}}{w_{Theoritial}} \times 100 = \frac{0.104}{0.173} \times 100 = 60\%$

47. The number of $>C = O$ groups present in a tripeptide Asp – Glu – Lys is

Ans. 5

Sol. Asp – Glu – Lys tripeptide is:



No. of CO group = 5

48. If 250 cm^3 of an aqueous solution containing 0.73 g of a protein A is isotonic with one litre of another aqueous solution containing 1.65 g of a protein B, at 298 K, the ratio of the molecular masses of A and B is_______ × 10^{-2} (to the nearest integer)

Ans. 177

Sol. For isotonic solution

$i_1C_1 = i_2C_2$ {For protein i = 1}

$C_1 = C_2$

$$\Rightarrow \quad \frac{0.73 \times 1000}{M_A \times 250} = \frac{1.65}{M_B \times 1}$$

$$\frac{M_A}{M_B} = \frac{0.73 \times 4}{1.65} = 1.77 = 177 \times 10^{-2}$$

49. The volume (in mL) of 0.1 N NaOH required to neutralise 10 mL of 0.1 N phosphinic acid is

Ans. 10

Sol. Phosphinic acid is hypo phosphorous acid (H_3PO_2).

$$NaOH + H_3PO_2 \longrightarrow NaH_2PO_2 + H_2O$$

For neutralisation

$(N_1V_1)_{acid} = (N_2V_2)_{base}$

$0.1 \times 10 = 0.1 \times (V_{mL})_{NaOH}$

$V_{NaOH} = 10$ mL

50. 0.023×10^{22} molecules are present in 10g of a substance 'x'. The molarity of a solution containing 5 g of substance 'x' in 2 L solution is__________$\times 10^{-3}$.

Ans. 25

Sol. Number of mol of X $= \frac{6.022 \times 10^{22}}{6.022 \times 10^{23}} = \frac{10}{\text{Molar mass of X}}$

So molar mass of X = 100g

$\text{Molarity} = \frac{5}{100 \times 2} = 0.025$ M

Ans. = 0.025 M

$M = 25 \times 10^{-3}$

So P = 25

PART-C : MATHEMATICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

51. Let A be a 3 × 3 matrix such that

$$\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix} \text{ and } B = \text{adj (adj A)}.$$

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to

(1) $\left(3, \frac{1}{81}\right)$ (2) $\left(9, \frac{1}{9}\right)$ (3) (3, 81) (4) $\left(9, \frac{1}{81}\right)$

Ans. (1)

Sol. $|\text{adj } A| = |A|^2 = 9$

$\Rightarrow$ $|A| = \pm 3 = \lambda$ $\Rightarrow$ $|\lambda| = 3$

$\Rightarrow$ $|B| = |\text{adj } A|^2 = 81$

$\Rightarrow$ $|(B^{-1})^T| = |B^{-1}| = |B|^{-1} = \frac{1}{|B|} = \frac{1}{81} = \mu$

52. If $x^3dy + xy.dx = x^2dy + 2y\,dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to :

(1) $\frac{\sqrt{e}}{2}$ (2) $\frac{1}{2} + \sqrt{e}$ (3) $\frac{3}{2}\sqrt{e}$ (4) $\frac{3}{2} + \sqrt{e}$

Ans. (3)

Sol. $x^3dy + xydx = 2ydx + x^2dy$

$\Rightarrow$ $(x^3 - x^2)dy = (2 - x)\,ydx$

$\Rightarrow$ $\frac{dy}{y} = \frac{2-x}{x^2(x-1)}dx$

$\Rightarrow$ $\int \frac{dy}{y} = \int \frac{2-x}{x^2(x-1)}dx$ .....(i)

Let $\frac{2-x}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$

$\Rightarrow$ $2 - x = A(x-1) + B(x-1) + Cx^2$

$\Rightarrow$ C = 1, B = –2 and A = –1

$\Rightarrow$ $\int \frac{dy}{y} = \int \left\{\frac{-1}{x} - \frac{2}{x^2} + \frac{1}{x-1}\right\}dx$

$\Rightarrow \quad \ell n\, y = -\ell n\, x + \frac{2}{x} + \ell n\,|x-1| + C$

$\therefore \quad y(2) = e$

$\Rightarrow \quad 1 = -\ell n\, 2 + 1 + 0 + C$

$\Rightarrow \quad C = \ell n\, 2$

$\Rightarrow \quad \ell n\, y = -\ell n\, x + \frac{2}{x} + \ell n\,|x-1| + \ell n\, 2$

at $\quad x = 4$

$\Rightarrow \quad \ell n\, y(4) = -\ell n\, 4 + \frac{1}{2} + \ell n\, 3 + \ell n\, 2$

$\Rightarrow \quad \ell n\, y(4) = \ell n\left(\frac{3}{2}\right) + \frac{1}{2} = \ell n\left(\frac{3}{2}e^{1/2}\right)$

$\Rightarrow \quad y(4) = \frac{3}{2}e^{1/2}$

53. If the sum of the series $20 + 19\frac{3}{5} + 19\cdot\frac{1}{5} + 18\frac{4}{5} + \ldots\ldots$ upto n^{th} term is 488 and the n^{th} term is negative, then :

(1) n^{th} term is $-4\frac{2}{5}$ (2) n = 41 (3) n^{th} term is – 4 (4) n = 60

Ans. (3)

Sol. $488 = \frac{n}{2}\left[2\left(\frac{100}{5}\right) + (n-1)\left(-\frac{2}{5}\right)\right]$

$488 = \frac{n}{2}(101 - n) \quad \Rightarrow \quad n^2 - 101n + 2440 = 0$

$\Rightarrow \quad n = 61 \;\text{ or }\; 40$

For n = 40 $\Rightarrow$ Tn > 0

For n = 61 $\Rightarrow$ Tn < 0

$T_n = \frac{100}{5} + (61-1)\left(-\frac{2}{5}\right) = -4$

54. The set of all real values of λ for which the quadratic equation $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is :

(1) (–3, –1) (2) (0, 2) (3) (1, 3] (4) (2, 4]

Ans. (3)

Sol. $f(0)\, f(1) \le 0$

$\Rightarrow \quad 2(\lambda^2 + 1 - 4\lambda + 2) \le 0 \quad \Rightarrow \quad 2(\lambda^2 - 4\lambda + 3) \le 0$

$(\lambda - 1)(\lambda - 3) \le 0$

$\Rightarrow \quad \lambda \in [1, 3]$

But at $\lambda = 1$, both roots are 1 so $\lambda \neq 1$

55. Ler R_1 and R_2 be two relations defined as follows :

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and $R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$,

where Q is the set of all rational numbers, then

(1) R_1 is transitive but R_2 is not transitive.
(2) R_2 is transitive but R_1 is not transitive.
(3) Neither R_1 nor R_2 is transitive
(4) R_1 and R_2 are both transitive.

Ans. (3)

Sol. For R_1 let $a = 1 + \sqrt{2}$, $b = 1 - \sqrt{2}$, $c = 81/4$

$aR_1b \Rightarrow a^2 + b^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in Q$

$aR_1c \Rightarrow b^2 + c^2 = (1 - \sqrt{2})^2 + (8^{1/4})^2 = 3 \in Q$

$aR_1c \Rightarrow a^2 + c^2 = (1 + \sqrt{2})^2 + (8^{1/4})^2 = 3 + 4\sqrt{2} \notin Q$

$\therefore$ R_1 is not transitive.

For R_2 let $a = 1 + \sqrt{2}$, $b = \sqrt{2}$, $c = 1 - \sqrt{2}$

$aR_2b \Rightarrow a^2 + b^2 = (1 + \sqrt{2})^2 + (\sqrt{2})^2 = 5 + 2\sqrt{2} \notin Q$

$bR_2b \Rightarrow b^2 + c^2 = (\sqrt{2})^2 + (1 - \sqrt{2})^2 = 5 - 2\sqrt{2} \notin Q$

$aR_2c \Rightarrow a^2 + c^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in Q$

$\therefore$ R_2 is not transitive.

56. The Plane which bisects the line joining the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point :

(1) (0, –1, 1)
(2) (4, 0, –1)
(3) (4, 0, 1)
(4) (0, 1, –1)

Ans. (2)

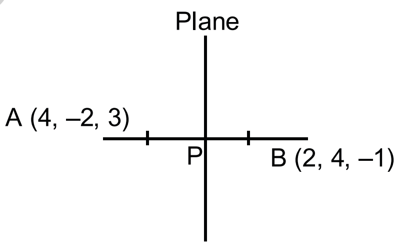
Sol. Mid point $P \equiv (3, 1, 1)$

Normal of plane is along the line AB.

D.R.'s of normal = 4 – 2, –2 –4, 3 –(–1) = 2, –6, 4 = 1, –3, 2

Plane $\rightarrow$ $1(x - 3) - 3(y - 1) + 2(z - 1) = 0$

$\Rightarrow$ $x - 3y + 2z - 2 = 0$



57. Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively :

(1) T, T, F
(2) T, T, T
(3) T, F, T
(4) F, T, F

Ans. (1)

Sol. $(p \wedge q)$ should be TRUE and $(\sim q \vee r)$ should be FALSE.

58. If a ΔABC has vertices A (–1, 7), B (–7, 1) and C (5, –5), then its orthocentre has coordinates:

(1) (– 3, 3)
(2) (3, – 3)
(3) $\left(-\frac{3}{5}, \frac{3}{5}\right)$
(4) $\left(\frac{3}{5}, -\frac{3}{5}\right)$

Ans. (1)

Sol.

$$m_{BC} = \frac{6}{-12} = -\frac{1}{2}$$

$\therefore$ Equation of AD is $y - 7 = 2(x + 1)$

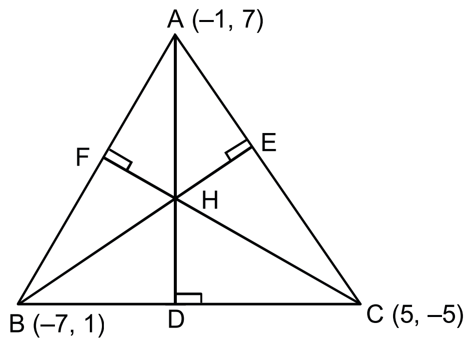
$y = 2x + 9$ ....(1)

$m_{AC} = \frac{12}{-6} = -2$

$\therefore$ Equation of BE is

$y - 1 = \frac{1}{2}(x + 7)$

$y = \frac{x}{2} + \frac{9}{2}$ ....(2)



by (1) and (2)

$2x + 9 = \frac{x+9}{2}$

$\Rightarrow 4x + 18 = x + 9$

$\Rightarrow 3x = 9 \Rightarrow x = -3$

$\therefore y = 3$

59. Suppose f(x) is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \{x \in R \mid f(x) = f(0)\}$,0 then the sum of squares of all the elements of T is :

(1) 4 (2) 6 (3) 2 (4) 8

Ans. (1)

Sol. $f'(x) = k.x(x + 1)(x - 1) = k(x^3 - x)$

$\Rightarrow \quad f(x) = k\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C$

$\Rightarrow \quad f(0) = C$

$\Rightarrow \quad f(x) = f(0)$

$\Rightarrow \quad k\frac{(x^4 - 2x^2)}{4} + C = C$

$\Rightarrow \quad x^2(x^2 - 2) = 0$

$\Rightarrow \quad x = 0, \sqrt{2}, -\sqrt{2}$

$\Rightarrow \quad T = \{0, \sqrt{2}, -\sqrt{2}\}$

60. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18k is equal to:

(1) 11 (2) 5 (3) 9 (4) 7

Ans. (4)

Sol. $T_{r+1} = {}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$= {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$

for the term independent of x put r = 6

$$\Rightarrow T_7 = {}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6 = {}^9C_3\left(\frac{1}{6}\right)^3 = \frac{9\times8\times7}{3\times2\times1}\left(\frac{1}{6}\right)^3 = \left(\frac{7}{18}\right)$$

61. If z_1, z_2 are complex numbers such that $Re(z_1) = |z_1 = 1|$ and $Re(z_2) = |z_2 = 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $Im(z_1 + z_2)$ is equal to :

(1) $2\sqrt{3}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{3}}$

Ans. (1)

Sol. $|z_1 - 1| = Re(z_1)$ Let z_1 $x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$(x_1 - 1)^2 + y_1^2 = x_1^2$

$y_1^2 - 2x_1 + 1 = 0$ ....(1)

$|z_2 - 1| = Re(z_2)$

$(x_2 - 1)^2 + y_2^2 = x_2^2$

$y_2^2 - 2x_2 + 1 = 0$ ......(2)

$y_1^2 - y_2^2 - 2(x_1 - x_2) = 0$

$(y_1 - y_2)(y_1 + y_2) = 2(x_1 - x_2)$

$y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right)$ .....(3)

$\arg(z_1 - z_2) = \frac{\pi}{6}$

$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{6}$

$\frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}}$ .....(4)

$\therefore$ $y_1 + y_2 = 2\sqrt{3}$ $\Rightarrow$ $Im(z_1 + z_2) = 2\sqrt{3}$

62. Let xi $(1 \leq i \leq 10)$ be ten observation of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \neq p \in R$, then the standard deviation of these observations is:

(1) $\frac{4}{5}$ (2) $\sqrt{\frac{3}{5}}$ (3) $\frac{9}{10}$ (4) $\frac{7}{10}$

Ans. (3)

Sol. $$S.D. = \sqrt{\frac{\sum_{i=1}^{10}(x_i - p)^2}{10} - \left(\frac{\sum_{i=1}^{10}(x_i - p)}{10}\right)^2}$$

$$\sqrt{\frac{9}{10}-\left(\frac{3}{10}\right)^2}=\frac{9}{10}$$

63. The probability that a randomly chosen 5-digit number is made from exactly two digits is:

(1) $\frac{135}{10^4}$ (2) $\frac{150}{10^4}$ (3) $\frac{134}{10^4}$ (4) $\frac{121}{10^4}$

Ans. (1)

Sol. total = $9(10^4)$

fav. way = ${}^9C_2\,(2^5-2)+{}^9C_1\,(2^4-1)=36(30)+9(15)=1080+135$

$$\text{Prob}=\frac{36\times 30+9\times 15}{9\times 10^4}=\frac{4\times 30+15}{10^4}=\frac{135}{10^4}$$

64. Let a, b, c $\in$ R be such that $a^2+b^2+c^2=1$. If $a\cos\theta=b\cos\left(\theta+\frac{2\pi}{3}\right)=c\cos\left(\theta+\frac{4\pi}{3}\right)$, where $\theta=\frac{\pi}{9}$, then the angle between the vectors $a\hat{i}+b\hat{j}+c\hat{k}$ and $b\hat{i}+c\hat{j}+a\hat{k}$ is:

(1) 0 (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{9}$

Ans. (3)

Sol. $a\cos\theta=b\cos\left(\theta+\frac{2\pi}{3}\right)=c\cos\left(\theta+\frac{4\pi}{3}\right)=k$

$$a=\frac{k}{\cos\theta},\ b=\frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)},\ c=\frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$$

$$ab+bc+ca=k^2\left[\frac{\cos\left(\theta+\frac{4\pi}{3}\right)+\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)}{\cos\left(\theta+\frac{4\pi}{3}\right)\cos\theta\cos\left(\theta+\frac{2\pi}{3}\right)}\right]$$

$$=k^2\left[\frac{\cos\theta+2\cos(\theta+\pi)\cdot\cos\left(\frac{\pi}{3}\right)}{\cos\theta\cdot\cos\left(\theta+\frac{2\pi}{3}\right)\cdot\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$$=k^2\left[\frac{\cos\theta-2\cos\theta\cdot\frac{1}{2}}{\cos\theta\cdot\cos\left(\theta+\frac{2\pi}{3}\right)\cdot\cos\left(\theta+\frac{4\pi}{3}\right)}\right]=0$$

$$\cos\phi=\frac{(a\hat{i}+b\hat{j}+c\hat{k})\cdot(b\hat{i}+c\hat{j}+a\hat{k})}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{b^2+c^2+a^2}}=ab+bc+ca=0$$

$$\phi=\frac{\pi}{2}$$

65. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec.), when the length of a side of the cube is 10 cm, is:

(1) 20 (2) 10 (3) 18 (4) 9

Ans. (4)

Sol. $S = 6a^2 \Rightarrow \quad \frac{ds}{dt} = 12a.\frac{da}{dt} = 3.6 \quad \Rightarrow \quad 12(10)\frac{da}{dt} = 3.6 \Rightarrow \quad \frac{da}{dt} = 0.03$

$V = a^3 \Rightarrow \quad \frac{dv}{dt} = 3a^2.\frac{da}{dt} = 3(10)^2.\left(\frac{3}{100}\right) = 9$

66. $\lim\limits_{x\to\infty} \frac{(a+2x)^{1/3} - (3x)^{1/3}}{(3a+x)^{1/3} - (4x)^{1/3}}$ $(a \neq 0)$ is equal to:

(1) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{1/3}$ (2) $\left(\frac{2}{3}\right)^{4/3}$ (3) $\left(\frac{2}{9}\right)^{4/3}$ (4) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{1/3}$

Ans. (4)

Sol. $\lim\limits_{x\to a} \frac{\frac{1}{3}(a+2x)^{-2/3}\cdot 2 - \frac{1}{3}\cdot(3x)^{-2/3}\cdot 3}{\frac{1}{3}(3a+2x)^{-2/3}\cdot - \frac{1}{3}(4x)^{-2/3}\cdot 4}$

$= \frac{\frac{1}{3}(3a)^{-2/3}\cdot(2-3)}{\frac{1}{3}(4a)^{-2/3}\cdot(1-4)} = \frac{3^{-2/3}}{4^{-2/3}}\cdot\frac{1}{3}$

$= \frac{2^{4/3}}{9^{1/3}}\cdot\frac{1}{3} = \frac{2}{3}\cdot\left(\frac{2}{9}\right)^{1/3}$

67. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1 (b < 5)$ and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to:

(1) (8, 10) (2) $\left(\frac{20}{3}, 12\right)$ (3) (8, 12) (4) $\left(\frac{24}{5}, 10\right)$

Ans. (1)

Sol. $e_1 = \sqrt{1 - \frac{b^2}{25}}$; $e_2 = \sqrt{1 + \frac{b^2}{16}}$

$e_1 e_2 = 1$

$\Rightarrow \quad (e_1 e_2)^2 = 1 \quad \Rightarrow \quad \left(1 - \frac{b^2}{25}\right)\left(1 + \frac{b^2}{16}\right) = 1 \quad \Rightarrow \quad 1 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^4}{25\times 16} = 1$

$\Rightarrow \quad \frac{9}{16\cdot 25}b^2 - \frac{b^4}{25\cdot 16} = 0 \quad \Rightarrow \quad b^2 = 9$

$e_1 = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

$e_2 = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

$\alpha = 2(5)(e_1) = 8$

$\beta = 2(4)(e_2) = 10$

$(\alpha, \beta) = (8, 10)$

68. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}(\sqrt{x}) + B(x) + C$, where C is a constant of integration, then the ordered pair (A(x), B(x)) can be :

(1) $\left(x-1, \sqrt{x}\right)$ (2) $\left(x-1, -\sqrt{x}\right)$ (3) $\left(x+1, \sqrt{x}\right)$ (4) $\left(x+1, -\sqrt{x}\right)$

Ans. (4)

Sol. $I = \int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)dx$

$\int \underset{I}{\tan^{-1}\left(\sqrt{x}\right)}\underset{II}{dx} = x\tan^{-1}\sqrt{x} - \int \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}\cdot xdx + C = x\tan^{-1}\sqrt{x} - \frac{1}{2}\int\frac{t.2t.dt}{1+t^2} + C\left(x = t^2\right)$

$= x\tan^{-1}\sqrt{x} - \int\frac{t^2}{1+t^2}dt + C = x\tan^{-1}\sqrt{x} - t + \tan^{-1}t + C = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$

$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C \quad \Rightarrow \quad (Ax) = x+1 \Rightarrow B(x) = -\sqrt{x}$

69. If the value of the integral $\int_0^{1/2}\frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to:

(1) $2\sqrt{3}+\pi$ (2) $2\sqrt{3}-\pi$ (3) $3\sqrt{2}+\pi$ (4) $3\sqrt{2}-\pi$

Ans. (2)

Sol. $\frac{k}{6} = \int_0^{\frac{1}{2}}\frac{x^2}{(1-x^2)^{3/2}}dx$ x = sinθ ; dx = cosθ dθ

$\Rightarrow \quad \frac{k}{6} = \int_0^{\frac{\pi}{6}}\frac{\sin^2\theta}{(1-\sin^2\theta)^{3/2}}\cdot\cos\theta d\theta \quad \Rightarrow \quad \frac{k}{6} = \int_0^{\frac{1}{2}}\frac{\sin^2\theta}{\cos^3\theta}\cdot\cos\theta d\theta$

$\Rightarrow \quad \frac{k}{6} = \int_0^{\frac{\pi}{6}}\tan^2\theta d\theta = \int_0^{\frac{\pi}{6}}(\sec^2\theta - 1)d\theta \quad \Rightarrow \quad \frac{k}{6} = (\tan\theta - \theta)_0^{\pi/6} = \left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right) = \frac{2\sqrt{3}-\pi}{6}$

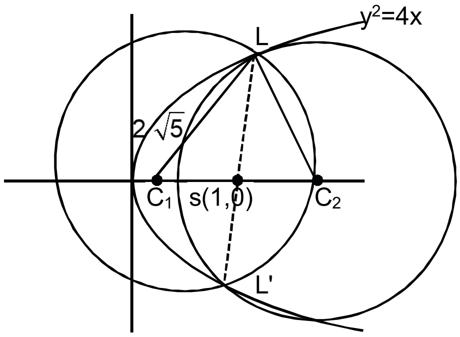
$\Rightarrow \quad k = 2\sqrt{3}-\pi$

70. Let the latus rectum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:

(1) 12 (2) 8 (3) $8\sqrt{5}$ (4) $4\sqrt{5}$

Ans. (2)

Sol.



$C_1 C_2 = 2C_1 S = 2\sqrt{20-4} = 8$

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

71. If the tangent to the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1,2) intersect at the same point on the x-axis, then the value of c is.....

Ans. 04.00

Sol. For (1,2) of $y^2 = 4x \Rightarrow t = 1, a = 1$

normal $\Rightarrow tx + y = 2at + at^3$

$\Rightarrow x + y = 3$ intersect x-axis at (3,0)

$y = e^x \quad \Rightarrow \frac{dy}{dx} = e^x$

tangent $\Rightarrow y - e^c = e^c (x - c)$

at (3,0) $\Rightarrow 0 - e^c = e^c (3 - c) \Rightarrow c = 4$

72. Let a plane P contain two lines

$\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \varepsilon R$ and

$\vec{r} = -\hat{i} + \mu(\hat{j} - \hat{k}), \mu \varepsilon R$.

If Q (α, β, γ) is the foot of the perpendicular drawn from the point M (1, 0, 1) to P, then $3(\alpha + \beta + \gamma)$ equals.......

Ans. 05.00

Sol. Normal of plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$\vec{n} = -\hat{i} + \hat{j} + \hat{k}$

D.R.'s = – 1, 1, 1

Plane $\Rightarrow \quad -1(x - 1) + 1(y - 0) + 1(z - 0) = 0$

$\Rightarrow \quad x - y - z - 1 = 0$

If (x, y, z) is foot of perpendicular of M(1, 0, 1) on the plane then

$$\Rightarrow \frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = \frac{-(1-0-1-1)}{3}$$

$$x = \frac{4}{3}, y = -\frac{1}{3}, z = \frac{2}{3}$$

$$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$$

73. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to :

Ans. 39.00

Sol. 3, A_1, A_2, A_3, ……..A_m, 243

$$d = \frac{243-3}{m+1} = \frac{240}{m+1}$$

3, G_1, G_2, G_3, 243

$$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{1/4} = 3$$

$G_2 = A_4$

$$\Rightarrow \quad 3(3)^2 = 3 + 4\left(\frac{240}{m+1}\right)$$

$$\Rightarrow \quad 27 = 3 + \frac{960}{m+1} \quad \Rightarrow \quad m + 1 = 40 \quad \Rightarrow \quad m = 39$$

74. The total number of 3-digit numbers, whose sum of digits is 10, is…….

Ans. 54.00

Sol. Let xyz be the three digit number

$x + y + z = 10,\ x \le 1,\ y \ge 0,\ z \ge 0$

$x - 1 = t \quad \Rightarrow \quad x = 1 + t \quad\quad x - 1 \ge 0$

$t \ge 0$

$t + y + z = 10 - 1$

$t + y + z = 9, \quad 0 \le t, z, z \le 9$

total number of non negative integral solution $= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = \frac{11.10}{2} = 55$

But for t = 9, x = 10, so required number of integers = 55–1 = 54.

75. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

Such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to….

Ans. 08.00

Sol. $x - 2y + 5z = 0$ …….(i)

$-2x + 4y + z = 0$ …….(ii)

$-7x + 14y + 9z = 0$ …….(iii)

$2 \times$ (i) + (ii) $\Rightarrow z = 0$

$\Rightarrow \quad x = 2y$

$\Rightarrow \quad 15 \le x^2 + y^2 + z^2 \le 150 \quad \Rightarrow \quad 15 \le 4y^2 + y^2 \le 150$

$\Rightarrow \quad 3 \le y^2 \le 30 \quad \Rightarrow \quad y = \pm 2, \pm 3, \pm 4, \pm 5 \quad \Rightarrow$ 8 solutions.